Image Registration using stationary velocity fields parameterized by norm-minimizing Wendland kernel

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Abstract

Interpolating kernels are crucial to solving a stationary velocity field (SVF) based image registration problem. This is because, velocity fields need to be computed in non-integer locations during integration. The regularity in the solution to the SVF registration problem is controlled by the regularity term. In a variational formulation, this term is traditionally expressed as a squared norm which is a scalar inner product of the interpolating kernels parameterizing the velocity fields. The minimization of this term using the standard spline interpolation kernels (linear or cubic) is only approximate because of the lack of a compatible norm. In this paper, we propose to replace such interpolations with a norm-minimizing – the Wendland kernel which has the same computational simplicity like B-Splines. An application on the Alzheimer’s disease neuroimaging initiative showed that Wendland SVF based measures separate Alzheimer’s disease vs. normal controls better than both B-Spline SVFs (p<0.05 in amygdala) and B-Spline freeform deformation (p<0.05 in amygdala and cortical gray matter).

Materials and Methods

Given a floating image \( I_f \) and a reference image \( I_r \), image registration involves finding a transformation \( ϕ \) that aligns these two images. This is achieved by formulating a cost function as follows:

\[
\arg\min_{ϕ} \| I_f - ϕ(I_r) \|^2 + λR(ϕ),
\]  

(1)

where \( λ \) is a user-specified constant, \( d \) is a dissimilarity measure that allows us to compare the floating image to the reference image and \( R \) is a regularization term that ensures \( ϕ \) is the simplest. Let \( Ω \) be the spatial domain of \( I_f \) with \( x \in \Omega \) as a spatial location. Let \( Diff(\Omega) \) be the space containing the diffeomorphic transformation \( ϕ: \Omega \rightarrow \Omega \). \( Ω \times \Omega \) is the tangent space of \( Diff(\Omega) \) at identity \( Id \) containing the velocity fields \( v \). The SVF \( v \) is then the unique solution of:

\[
\frac{∂N(x, t)}{∂t} = v(ϕ(x, t)), \quad v = ϕ(x, 1) = \exp(V),
\]  

(2)

Traditionally, the regularization term in (1) is chosen to be the norm induced by a differential operator \( L, R = \| L[N] \| \), where \( [\cdot] \) is a 2-norm. With certain conditions on \( L \), there exists a Hilbert space \( V \) with norm \( \| \cdot \|_V \) so that \( [\cdot]_V = \| L[N] \| \), \( ϕ \in \text{Diff}(\Omega) \) is the inner product. In this paper we take a different approach. There exists reproducing kernels \( K : \Omega \times \Omega \rightarrow \mathbb{R}^{d_0} \), which induce a norm \( [\cdot]_K \) for which there need not be a corresponding differential operator. The reproducing kernels have the computational advantage that their inner product may be calculated which induce a norm theoretically founded variational diffeomorphic registration scheme using SVFs parametrized by norm-minimizing Wendland kernels. Performance-wise, with respect to atrophy scoring, we showed that this scheme performs on par or better than B-Spline based FFDs.

Results

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Conclusions

We have proposed a novel variational formulation for flow-based diffeomorphism registration scheme. By combining the RKHS property and efficiency of SVFs, we have constructed an efficient and theoretically founded variational diffeomorphic registration scheme using SVFs parameterized by norm-minimizing Wendland kernels. Performance-wise, with respect to atrophy scoring, we showed that this scheme performs on par or better than B-Spline-based FFDs and SVFs.

References


Figure 1: A graphical representation of a 1-d Wendland kernel with a support of 2 and a coefficient of 1.

Material 1734 to 2301

The velocity field defined in (2) may now be parameterized using Wendland kernels as follows:

\[
\sum_{i,j,k} (K_i(x,y) - b_i b_j K_{i,j}(x,y)) = \sum_{i,j,k} (K_i(x,y) - b_i b_j K_{i,j}(x,y)),
\]  

(3)

Example of reproducing kernel with finite support are the Wendland kernels [3]. They were originally developed for multi-dimensional, scattered grid interpolation. They are positive definite functions of minimal degree (smooth) radial basis functions on \( \mathbb{R}^d \). They are defined as follows, where \( k \) is the smoothness of the kernel, \( s \) is the dimension given by \( \lceil \frac{d+1}{k} \rceil \), and \( x' \) is the integral operator applied t times. We choose \( t = 3 \) and \( k = s-1 \), since \( k \) yields a \( C^k \) smooth Wendland kernel. It evaluates to,

\[
ϕ(x, t) = (1 - t)^{s-1} \frac{t}{s-1}.
\]  

(4)

Table 1: Various statistics on atrophy estimated using Wendland based and B-spline based registration schemes; mean and standard deviation are in WHV. Whole Brain, Hap: Hippocampus, MTL: Medial Temporal Lobe, Vent: Ventricles, CGM: Cortical gray matter and Amy: Amygdala. The p-values are from a null hypothesis test for equal measures between B-spline based scores and Wendland based scores. * indicates significance (p<0.05) when compared to B-Spline SVFs and † indicates significance (p<0.05) when compared to B-Spline FFDs. It implies significance when compared to B-Spline FFD after bonferroni correction.

Conclusions

We have proposed a novel variational formulation for flow-based diffeomorphism registration scheme. By combining the RKHS property and efficiency of SVFs, we have constructed an efficient and theoretically founded variational diffeomorphic registration scheme using SVFs parameterized by norm-minimizing Wendland kernels. Performance-wise, with respect to atrophy scoring, we showed that this scheme performs on par or better than B-Spline based FFDs and SVFs.