Image Registration using stationary velocity fields parameterized by norm-minimizing Wendland kernel

Akshay Pai, Stefan Sommer, Lauge Srensen, Jon Sporring and Mads Nielsen

Department of Computer Science, University of Copenhagen

UNIVERSITY OF COPENHAGEN

Abstract

Interpolating kernels are crucial to solving a stationary velocity field (SVF) based image registration problem. This is because, velocity fields need to be computed in non-integer locations during integration. The regularity in the solution to the SVF registration problem is controlled by the regularization term. In a variational formulation, this term is traditionally expressed as a squared norm which is a scalar inner product of the interpolating kernels parameterizing the velocity fields. The minimization of this term using the standard spline interpolation kernels (linear or cubic) is only approximative because of the lack of a compatible norm. In this paper, we propose to replace such interpolants with a norm-minimizing interpolant - the Wendland kernel which has the same computational simplicity like B-Splines. An application on the Alzheimer's disease neuroimaging initiative showed that Wendland SVF based measures separate (Alzheimer's disease v/s normal controls) better than both B-Spline SVFs (p<0.05 in amygdala) and B-Spline freeform deformation (p<0.05 in amygdala and cortical gray matter).

Materials and Methods

Given a floating image I_1 and a reference image I_2, image registration involves finding a transformation φ that aligns these two images. This is achieved by formulating a cost function as follows:

$$\arg\min_{\varphi}(E(I_1, I_2)) = \arg\min_{\varphi} d(I_1(\varphi), I_2) + \lambda R(\varphi), \tag{1}$$

where λ is user-specified constant, d is a dissimilarity measure that allows us to compare the floating image to the reference image and R is a regularization term that ensures φ is the simplest. Let Ω be the spatial domain of I_1 with $\mathbf{x} \in \Omega$ as a spatial location. Let $\text{Diff}(\Omega)$ be the space containing the diffeomorphic transformation $\varphi : \Omega \to \Omega$, $\phi : \Omega \times \mathbb{R} \to \Omega$ and finally, V be the tangent space of $\text{Diff}(\Omega)$ at identity Id containing the velocity fields $\mathbf{v}$. The SVF $\mathbf{v}$ is then the unique solution of:

$$\begin{aligned}\frac{\partial \phi(\mathbf{x}, t)}{\partial t} &= \mathbf{v}(\phi(\mathbf{x}, t)),\\ \varphi &= \phi(\mathbf{x}, 1) = \text{Exp}(\mathbf{v}),\end{aligned} \tag{2}$$

Traditionally, the regularization term in (1) is chosen to be the norm induced by a differential operator $\mathcal{L}$, $R = ||\mathcal{L}\mathbf{v}||_V^2$, where $||\cdot||$ is a 2-norm. With certain conditions on $\mathcal{L}$, there exists a complete Hilbert space $V \subset L_2$ with norm $||\cdot||_V$ so that $||\mathcal{L}\mathbf{v}||^2 = ||\mathbf{v}||_V^2 = \langle \mathbf{v}, \mathbf{v}\rangle_V$, where $\langle\cdot,\cdot\rangle_V$ is the inner product.

In this paper we take a different approach. There exists reproducing kernels $K : \Omega \times \Omega \to \mathbb{R}^{d\times d}$, which induce a norm $||\cdot||_W$ for which there need not be a corresponding differential operator. The reproducing kernels have the computational advantage that their inner product may be calculated directly as

$$\langle \mathbf{a}, \mathbf{b}\rangle_W = a^T K(\mathbf{x}, \mathbf{y})\mathbf{b}$$

for all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^d$, for all kernel centers $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$, and where the inner product is taken over the dimension indicated by the dot. The space W is called a reproducible kernel Hilbert space (RKHS).

When W is induced from a differential operator $\mathcal{L}$, the reproducing kernels are the Green's functions of $\mathcal{L}^*\mathcal{L}$ where $\mathcal{L}^*$ is the complex conjugate of $\mathcal{L}$. Conversely, if we can choose a reproducing kernel, we know there is an associated RKHS as a completion of the space of functions spanned by the elements of the kernel. The norm and inner product on this space are defined from the kernel. We approximate this space by $\sum_i K(\cdot, \mathbf{x}_i)\mathbf{a}_i$, and by linearity of the inner product and the reproducing property, the norm on linear combinations of the kernel can be evaluated by

$$\begin{aligned}\left\|\sum_i K(\cdot, \mathbf{x}_i)\mathbf{a}_i\right\|^2 &= \left\langle \sum_i K(\cdot, \mathbf{x}_i)\mathbf{a}_i, \sum_j K(\cdot, \mathbf{x}_j)\mathbf{a}_j \right\rangle\\ &= \sum_{i,j} \langle K(\cdot, \mathbf{x}_i)\mathbf{a}_i, K(\cdot, \mathbf{x}_j)\mathbf{a}_j\rangle\\ &= \sum_{i,j} \mathbf{a}_i^T K(\mathbf{x}_i, \mathbf{x}_j)\mathbf{a}_j.\end{aligned} \tag{3}$$

Example of reproducing kernel with finite support are the Wendland kernels [3]. They were originally developed for multi-dimensional, scattered grid interpolation. They are positive definite functions with positive Fourier transforms. They are minimal degree polynomials on $[0, 1]$ and yield C^{2k} (k is the desired degree of smoothness) smooth radial basis functions on $\mathbb{R}^d$. They are defined as follows, where k is the smoothness of the kernel, v is the dimension given by $\lfloor d/2 \rfloor + k + 1$ and $\mathcal{I}^k$ is the integral operator applied k times. We choose $d = 3$ and $k = 1$, since it yields a C^2 smooth Wendland kernel. It evaluates to,

$$\psi(r)_{3,1} = (1 - r)_+^4(4r + 1). \tag{4}$$

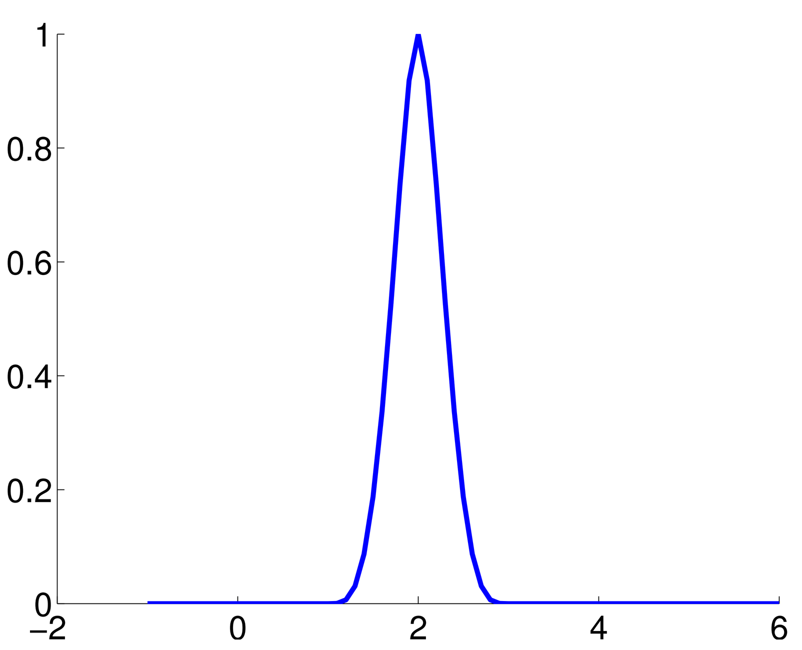

Figure 1: A graphical representation of a 1-d wendland kernel with a support of 2 and a coefficient of 1

The velocity field defined in (2) may now be parameterized using Wendland kernels as follows,

$$\mathbf{v}(\mathbf{x_j}) = \sum_i^N \mathbf{p}_i \psi(r(\mathbf{x}_i, \mathbf{x}_j)). \tag{5}$$

Note that $r = \frac{||\mathbf{x_i} - \mathbf{x_j}||_2}{a}$, where a is the scaling parameter, $\mathbf{p}_i \in \mathbb{R}^d$ is the coefficient attached with every kernel center $\mathbf{x}_i$, and N is the number of kernels having an influence on $\mathbf{x}$. The final cost function we optimize for is therefore,

$$\arg\min_{\mathbf{v}}(E(I_1, I_2)) = \arg\min_{\mathbf{v}} d_1(I_1(\mathbf{Exp}(\mathbf{v})), I_2) + \lambda \sum_{i,j} \mathbf{p}_i^T \psi(r(\mathbf{x}_i, \mathbf{x}_j))\mathbf{p}_j. \tag{6}$$

Data

Data used in the preparation of this article was obtained from the Alzheimer's Disease Neuroimaging Initiative (ADNI) database (http://www.adni-info.org/). Baseline and 12 month follow-up 1.5T 3DT1-weighted scans were analyzed for a subset of patients from the ADNI database. This included 24 NC and 48 AD patients [1]. Freesurfer pipeline [2] was used to correct for bias and generate segmentations. The images are given in 256^3 isotropic voxel cubes, where each voxel is 1 mm^3 cube. The segmentations are used to define the ROI.

Results

		AD	NC		
		mean (std)	mean (std)	AUC	Cohen's D
Wendland SVFs	WB	-1.67(1.12)	-0.49(1.11)	0.76(0.06)	1.06
	Hip	-3.38(2.50)	-0.27(1.62)	0.85(0.04)	1.47
	Vent	11.65(6.48)	4.50(4.91)	0.82(0.05)	1.24
	MTL	-3.12(2.10)	-0.54(1.30)	0.86(0.05)	1.48
	CGM	-2.56(1.65)	-0.41(0.97)	$\mathbf{0.88}^{*}$(0.04)	$\mathbf{1.58}^{*}$
	Amy	-3.21(4.16)	-1.34(2.57)	$\mathbf{0.70}^{*\#}$(0.06)	$\mathbf{0.54}^{*\#\dagger}$
B-spline SVFs	WB	-1.78(1.23)	-0.52(1.23)	0.74(0.06)	1.02
	Hip	-4.14(3.26)	-0.41(1.89)	0.85(0.04)	1.40
	Vent	11.95(6.60)	4.56(5.09)	0.82(0.05)	1.25
	MTL	-3.48(2.19)	-0.62(1.37)	0.87(0.04)	1.56
	CGM	-2.45(1.91)	-0.27(1.29)	0.84(0.05)	1.34
	Amy	-2.19(6.18)	-1.54(3.52)	0.56(0.07)	0.13
B-spline FFDs	WB	-1.22(0.90)	-0.37(0.68)	0.76(0.06)	1.07
	Hip	-3.37(2.01)	-0.83(1.43)	0.85(0.04)	1.46
	Vent	10.98(6.15)	4.39(4.84)	0.81(0.05)	1.19
	MTL	-2.82(1.93)	-0.78(1.26)	0.85(0.04)	1.25
	CGM	-2.07(1.80)	-0.51(1.10)	0.78(0.05)	1.04
	Amy	-1.83(6.10)	-1.48(3.56)	0.57(0.07)	0.07

Table 1: Various statistics based on atrophy estimated using Wendland based and B-spline based registration schemes; mean and standard deviation are in %WB: Whole Brain, Hip: Hippocampus, MTL: Medial Temporal Lobe, Vent: Ventricles, CGM: Cortical gray matter and Amy: Amygdala. The p-values are from a null hypothesis test for equal measures between B-spline based scores and Wendland based scores;$*$ indicates significance (p<0.05) when compared to B-Spline SVFs and # indicates significance (p<0.05) when compared to B-Spline FFDs.† implies significance when compared to B-Spline FFD after bonferroni correction

Conclusions

We have proposed a novel variational formulation for flow-based diffeormorphic registration scheme. By combining the RKHS property and efficiency of SVFs, we have constructed an efficient and theoretically founded variational diffeomorphic registration scheme using SVFs parametrized by norm-minimizing Wendland kernels. Performance-wise, with respect to atrophy scoring, we showed that this scheme performs on par or better than B-Spline based FFDs and SVFs.

References

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